

Mathematics Applications Unit 3/4

Test 5 2022

Calculator Assumed
Finance

STUDENT'S NAME

Solutions

DATE: Wednesday 10th August

TIME: 50 minutes

MARKS: 50

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser
 Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

1. (4 marks)

A sculpture was sold for \$22 000 by an art gallery.

- (a) The sculpture could be bought on terms in which a premium of 8% was added to the purchase price and then the total amount repaid to the gallery in 12 equal monthly repayments. How much would each repayment be? [2]

$$\begin{aligned}
 A &= \$22\,000 \times 1.08 \\
 &= \$23\,760 \checkmark \text{ annual amount} \\
 \text{mthly payment} &= \$23\,760 \div 12 \\
 &= \$1\,980 / \text{mth.} \checkmark \text{ mthly payment.}
 \end{aligned}$$

- (b) The sculpture was expected to increase in value at an annual rate of 9%. What is the expected value of the sculpture three years after it was purchased, to the nearest one hundred dollars? [2]

$$\begin{aligned}
 V &= 22\,000 \times 1.09^3 \\
 &= \$28\,490.64 \checkmark \text{ correct calculation} \\
 &= \$28\,500 \checkmark \text{ rounded to nearest 100.}
 \end{aligned}$$

2. (7 marks)

- (a) Agnetha and Bjorn are saving the same amount of money each week towards buying a home. State which of the following three investment options would grow their savings at the fastest rate and explain how it is possible to tell without having to calculate anything. [2]

A: An account offering 5% interest per annum, compounded monthly.

B: An account offering 5% simple interest per annum.

C: An account offering 5% interest per annum, compounded annually.

Option A ✓

correct option

All options offer 5% pa.

Option A is compounded more

often ∴ will attract more interest ✓ mentions compounding

- (b) An amount of \$22 000 can also be invested using two different saving schemes: Scheme A, in which interest of 5.44% per annum is compounded monthly; and Scheme B, in which interest of 5.46% per annum is compounded quarterly.

- (i) Calculate the effective interest rates of both schemes, correct to 4 decimal places, and hence state which scheme would pay the most interest over three years. [3]

A: 5.5777% pa ✓

B: 5.5728% pa ✓

∴ scheme A ✓
as its interest rate is higher than

correct value

- (ii) Calculate the interest that accrues over three years on \$22 000 with a rate of 5.45% per annum, compounded every four months. [2]

$$A = 22000 \left(1 + \frac{0.0545}{3} \right)^{3 \times 3}$$

$$= \$25869.77 \checkmark$$

total amount

$$\text{Int} = \$25869.77 - 22000$$

$$= \$3869.77 \checkmark$$

interest amount

3. (4 marks)

Trinity College would like to award a \$5 200 scholarship for the Mathematics Applications award winner each year. They set up a fund designed to pay the award winner the interest in the fund each year.

(a) What type of fund did they set up?

[1]

perpetuity ✓

(b) If the account earns interest at a rate of 5.5% p.a compounded monthly, determine the amount that Trinity needs to invest in the fund to ensure they can pay the scholarship each year.

[3]

$$P + 5200 = P \left(1 + \frac{0.055}{12} \right)^{12}$$

✓ partially correct
✓ All correct substitution

$$P = \$92185.73 \quad \checkmark \text{ correct amount.}$$

4. (6 marks)

Elton purchases a new grand piano for \$26 800. For the first two years the piano depreciates at a rate of 14% per year and for the third year it depreciates at a lower rate of 11.2% per year.

(a) Calculate the value of the piano after one year. [1]

$$V = 26800(1 - 0.14)^1 \\ = \$23048. \checkmark$$

(b) Calculate the value of the piano after the first three years. [2]

$$V_2 = 26800(1 - 0.14)^2 \\ = \$19821.28 \checkmark \\ V_3 = \$19821.28(1 - 0.112)^1 \\ = \$17601.30 \checkmark$$

balance after
2 years.

balance
after 3 years

For the next three years, the rate of depreciation is constant at $r\%$ per year. The average rate of depreciation for the first six years is 11% per year.

(c) Calculate the value of r as a percentage. [3]

$$\text{Value after } 6 \text{ yrs} = \$26800(1 - 0.11)^6 \\ = \$13319.10. \checkmark$$

correct value
after 6 yrs.

$$13319.10 = 17601.30(1 - r)^3 \\ r = 0.0887.$$

correct use
of formula

$$\therefore r = 8.87\% \text{ p.a.}$$

correct value
of r as a
%.

5. (11 marks)

Benny has retired and invested his lump sum superannuation payout of \$812 846 at a rate of 4.9% per annum compounded monthly. He begins the investment strategy from 1 January.

(a) Benny will receive \$4 900 at the end of each month for general living expenses and will also receive a further \$4 200 at the end of each year for an annual holiday.

(i) Identify this type of account. [1]

Annuity ✓ correct answer.

(ii) Determine the balance in the account at the end of the first year, [4]

4 entries correct.

N: 12 P/Y 12
I: 4.9 C/Y 12

$$\text{Bal} = \$793\,443.55 - \$4\,200$$

PV: -812846

$$\text{End 1st yr} = \$78\,9243.55$$

uses only \$4900.

PMT: 4900

FV: \$793433.55 ✓ F.V. correct

✓ final balance

(iii) Determine the balance in the account at the end of the second year, [3]

N: 12 P/Y: 12

$$\text{Bal} = \$768\,658.25$$

I: 4.9 C/Y: 12

PV: -789243.55

$$\text{End 2nd yr} = \$768\,658.25 - \$4\,200$$

PMT: \$4900

$$= \$76\,4458.25$$

FV: \$768658.25 ✓ F.V.

correct balance.

(b) When Benny retired, he also considered the option of setting up a perpetuity with his superannuation payout still at 4.9 % per annum compounded monthly. Calculate the quarterly payments Benny would have received with this perpetuity in place [3]

N: 12

I: 4.9

PV: -812846

$$\text{payment} = \$9998.09 / \text{qtr.}$$

PMT: \$9998.09

FV: 812846

P/Y: 4 ✓ correct P/Y

C/Y: 12

correct payment.

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PMT: \$9998.09

correct payment.

FV: 812846

P/Y: 4 ✓ correct P/Y

C/Y: 12

✓ entries correct.

6. (18 marks)

Freda opened her new music shop and organised a loan of \$18 000 to purchase new instruments. To pay off the loan, Freda makes repayments of \$650 at the end of each month. The spreadsheet below shows the progress of her loan.

Month	Amount owing at start of the month (\$)	Interest (\$)	Repayment (\$)	Amount owing at end of month (\$)
1	18 000.00	144.00	650	17 494.00
2	17 494.00	139.95	650	16 983.95
3	16 983.95	135.87	650	16 469.82

(a) Write a calculation to show that the yearly interest rate is 9.6%. [2]

$$\frac{144}{18000} \times 100 = 0.008 \quad \checkmark \text{ correct monthly}$$

$$0.008 \times 12 = 9.6\% \text{ pa} \quad \checkmark \text{ correct annual.}$$

(b) Complete the third row of the spreadsheet. [3]

\checkmark correct opening balance
 \checkmark correct amount of interest.
 \checkmark closing balance.

(c) Write a recursive rule to determine the closing balance of the loan at the end of each month. [2]

$$T_{n+1} = T_n \left(1 + \frac{0.096}{12} \right) \quad T_0 = 18000.$$

\checkmark rule. \checkmark T_0

(d) How much does she owe on the loan after the first year? [1]

$\$11653.58 \quad \checkmark$
 correct value (make for \$11654)

(e) Determine how many months it will take Freda to pay off the loan under these conditions. [1]

32 mths \checkmark correct no. of mths.

- (f) Calculate how much interest will she pay over the duration of the loan. ✓ calculates int. [3]

$$\begin{aligned} \text{Amount paid} &= (31 \times 650) + (277.60 \times 1.008) \\ &= 20150 + 279.82 \\ &= \$20429.82 \end{aligned}$$

✓ subtracts loan amount

$$\text{Int} = \$20429.82 - 18000 = 2429.82$$

ans.

After the first year of repaying the loan, Freda's working conditions change and due to earning less money she needs to make some changes to her loan terms going forward. Freda negotiates an interest only loan for 12 months, this means that she will repay the interest earned and not reduce the balance of the loan for the next 12 months.

- (g) Determine what her monthly repayment will be for the next 12 months (the second year of the loan) and what the balance of her loan will be at the end of the second year? [2]

$$\text{Balance after 12 mths} = \$11653.58$$

✓ balance after 12 mths

$$\text{payments} = \$93.23$$

✓ correct mthly payment.

At the start of the third year Freda is able to increase her monthly repayment to \$500 and she continues this until the loan is fully repaid.

- (h) What effect will all the changes to her loan conditions have on the duration of the loan and the total cost of the loan? Show your calculations to justify your answer. [4]

$$\text{Start 3rd year} = \$11653.58$$

$$N: 25.90$$

✓ correct time

$$I: 9.6$$

$$PV: -11653.58$$

$$PMT: 500$$

$$FV: 0$$

$$P/Y: 12$$

$$C/Y: 12$$

$$T_{\text{inc}} = 26 + 12 + 12$$

$$= 50 \text{ mths}$$

✓ final time.

increase from 32 mths.

$$\text{Int (1} \rightarrow \text{12) yr} = \$1453.58$$

$$\text{2nd yr (13} \rightarrow \text{24)} = 12 \times 93.23 = \$1118.76$$

$$\text{3rd yr} \rightarrow 50 \text{ mths} = 91295.42$$

✓

$$\text{Total int} = \$3867.76$$

✓

inc increase from \$2152.23